Performance Modelling for Data Monitoring Services in Smart Grid: A Network Calculus-Based Approach

Junwei Cao, *Senior Member, IEEE*, Yuxin Wan, Haochen Hua, *Member, IEEE*, and Gang Yang

***Abstract*—This paper focuses on solving the modelling issues of the monitoring system service performance based on network calculus theory. Firstly, we formulate the service model of the smart grid monitoring system. Then, we derive the flow arrival curve based on the incremental process related functions. Next, flow arrival curves for the case of the incremental process being fractional Gaussian process and that of generalized Cauchy process are obtained, respectively. Three technical theorems related to network calculus are presented as our main results. Mathematically, the variance of arrival flow for the continuous time case is derived. Assuming that the incremental process of network flow is a Gaussian stationary process, and given the auto-correlation function of incremental process with violation probability, the formula of the arrival curve is derived. Besides, the overall flow variance under the discrete time case is derived explicitly. The theoretical results are evaluated in smart grid applications. Simulations indicate that generalized Cauchy process performs better than fractional Gaussian process for our considered problem.**

***Index Terms*—fractional Gaussian process, generalized Cauchy process, monitoring systems, network calculus, service performance.**

I. Introduction

A. Motivation

THE monitoring system is one of the core elements within the field of smart grid [1]. Allowing for the monitoring of voltage, current, power at transformers, smart meters and distribution switching devices, the smart grid monitoring system has attracted much attention and significant advances on this topic have been made; see, e.g., [2], [3]. There are two main differences between the smart grid monitoring system and the traditional network service system. The analysis and management part of the smart grid monitoring system cannot be merely regarded as simple data transmission. In addition, the monitoring system is often required to analyze the synchronization of different monitoring node data. In the case of the power quality analysis, e.g., in [4], one needs to deal with the monitoring data acquired by power quality monitor simultaneously.

Currently, there have been two sets of the wide area data measurement systems, one of which is supervisory control and data acquisition (SCADA) system based on the remote terminal unit (RTU) [5], [6], and the other is the wide area measurement system (WAMS) based on the phasor measurement unit (PMU) [7], [8]. Based on RTU [9], the SCADA system is most widely used in power systems. The RTU has functions including measurement, communication, control, etc., and it is widely used in the energy management system. The main disadvantage of the RTU is that the data sampling frequency is relatively low, thus the dynamic information of the power communication network cannot be achieved in time. Moreover, the RTU has no synchronization clock, and the obtained data is not synchronized. If we compare PMU with RTU, PMU has several advantages as follows. It is equipped with global positioning system (GPS) based clock synchronizer and has higher measurement accuracy. Additionally, its measurement frequency is in the magnitude of tens of milliseconds. Therefore, compared with SCADA system, WAMS based on PMU can achieve more accurate measurement information and is the main component of the wide area monitoring system of smart grid [8]. The WAMS of smart grid is composed of three parts: PMU, communication network and controller. PMU measures the operation parameters of the grid from different regions and sends them to the control center where data is analyzed and processed. To illustrate, Fig. 1 is a schematic diagram for the case of wide area closed loop damping control in the South China Power Grid.

Under the mode of the shared network, the smart grid network monitoring system is required to analyze the service performance of the system, so as to determine the network configuration of the monitoring system. On the other hand, the performance analysis is based on the modelling of the network transmission and calculation performance of the monitoring system. Within the monitoring system of smart grid, both power measurement and controller signals are transmitted through the communication network, while the data of each node of the power system enters into the control center through different communication channels. In addition, the arrival time of data from different communication channels entering into the control center is different. Thus, it results that the input

Junwei Cao, Yuxin Wan, Haochen Hua (corresponding author, e-mail: hhua@tsinghua.edu.cn) and Gang Yang are with Beijing National Research Center for Information Science and Technology, Tsinghua University, Beijing, P. R. China.

control data is not synchronized. The data which is not synchronized with the communication channel brings additional delay into the system.

The above analysis indicates that the transmission model of the smart grid monitoring system cannot be simply equated with the ordinary network transmission model. The additional time delay caused by the synchronization is required to be considered. Besides, it costs time for the smart grid monitoring system to process the data. Thus, the time delay caused by the processing is also required to be considered in the service model; see, e.g., [10].

Fig. 1. Schematic diagram for the wide area monitoring system of smart grid.

B. Literature Review

The most commonly used methodology for the network performance analysis is based on the time delay generated by Ping and other network commands. Then, the network performance analysis can be expressed by the mean and variance of the time delay [11], [12]. Theoretically, the measurement is under the sampling of a continuous time model. If the sampled object is a group of independent and identically distributed random variables, and only if the random variables have the same and finite mathematical expectation (mean) μ and variance σ^2, the measured data shall be restricted by the central limit theorem, which makes the sampled data follow a normal distribution with mean μ and variance σ^2 [13], [14].

Paradoxically, the data of the Internet has been proved to be neither independent, nor does it follow the same distribution, with its variance unfixed [15]. For example, the variance of the fractional Brownian process is related to the size of the time interval. Even if the size of the sampling time interval is reduced, according to the fractional theory and the self-similarity theory, the random process that has fractional characteristics still has a burst phenomenon in a short period, even when the time interval is approaching zero. Therefore, in order to describe the performance of the network correctly, the method based on the mean and variance of the measured data is inaccurate.

The network calculus theory has been developed rapidly; see, e.g., [13]-[16], and it has been utilized to describe the performance of the service system [17], [18]. On the other hand, the future smart grid is expected to be an interconnected network of small-scale and self-contained microgrids, where renewable energy sources play a significant role [19], [20]. The stochastic network calculus theory has been applied to analyze the power supply reliability with various renewable energy configurations [21], [22]. In [23], network calculous theory has been applied in real-time routing in wireless sensor networks with the methodology of potential field in physics. For the survey of deterministic and stochastic service curve models in network calculus, readers can refer to [24]. It is notable that due to the synchronous property of the smart grid applications [25], the original network calculus theory cannot be directly applied in the delay analysis discussed in this paper.

C. Contribution

In this paper, network calculus theory has been applied to model the smart grid monitoring system. Firstly, the service model of the smart grid monitoring system is formulated. Then, the flow arrival curve based on the incremental process related functions is derived. Next, flow arrival curve for the case of the incremental process being fractional Gaussian process and generalized Cauchy process are obtained, respectively. Then, simulations are illustrated to show the feasibility of the proposed methods.

The main contributions of this work can be summarized as follows:

1, For the smart grid monitoring system, the network calculus theory has been applied, such that the engineering practical problems are transformed into mathematical issues. It is highlighted that we are focusing on developing fundamental as well as theoretical results, and the analysis methods introduced in this paper can be applied under most scenarios of monitoring systems in the field of smart grid.

2, Based on the network calculus theory, in this paper, a service modelling for the smart grid monitoring system is formulated. The flow arrival curve based on the incremental process related functions is derived. Flow arrival curves for the case of the incremental process being fractional Gaussian process and that of generalized Cauchy process are obtained, respectively. With the performance modelling method proposed in this paper, the smart grid network monitoring system shall be able to analyze the service performance of the system, so as to determine the network configuration of the monitoring system.

3, In this paper, three theorems are proposed as our main results. In Theorem I, the variance of arrival flow for the continuous time case is derived. In Theorem II, assuming that the incremental process of network flow is a Gaussian stationary process, and given the auto-correlation function of incremental process with violation probability, the formula of the arrival curve is derived. In Theorem III, the overall flow variance under the discrete time case is expressed explicitly. Besides, several technical lemmas and corollaries are obtained.

4, Based on the actual network flow dataset, numerical simulations are performed, verifying the feasibility of the obtained arrival flow curves. In addition, we compare the calculation results obtained under the assumption of fractional Gaussian process and generalized Cauchy process. The experimental results show that performance under generalized Cauchy process is closer to statistical results from real data.

The rest of this paper is organized as follows. In Section II, system modelling is introduced. In Section III, main results of this paper are provided. Experimental results are illustrated in Section IV. Finally, Section V gives some concluding remarks.

II. SERVICE MODELLING OF THE SMART GRID MONITORING SYSTEM

We regard the transmission network of the smart grid monitoring system as a transport service node. The processing and queuing of the equivalent transmission service node causes the delay of network transmission. Similarly, we treat the calculation and procession of the control center as a calculation service node. Since the equivalent transmission model of network services has been analyzed in detail in the network calculus theory, the model of the equivalent calculation service node of the monitoring system is given as follows. Data processing can be considered as a data stream going into a service link, with the output flow through a scaling function. Equivalently, the data is processed through a scaling function first, and then through a computing service. The latter case is consistent with the model described in [26], and we analyze the latter case here.

First, let us introduce some notations. Denote Δ as the processing time of one unit of data in one node. $F(\cdot)$ stands for the function to measure the time complexity. For n units of input data, their processing time is the value of $F(n)\Delta$. Let us define $C(t) \triangleq \frac{F^{-1}t}{\Delta}$, where $C(t)$ is considered as a strict service curve, whose definition can be found in network calculus theory [13]-[16]. Furthermore, when the processing result of the calculation processing node is expressed as the scaling function $S(n)$ of the input data with volume n, then the monitoring system can be regarded as a series of service models shown in Fig. 2. Thereby, the overall service performance of the monitoring system can be expressed as $C\otimes\underline{S^{-1}}(\beta)$, where $\underline{S(n)}$ stands for the infimum of scaling function, $\beta(t)$ stands for the equivalent transmission service performance of the monitoring system, and operator $\otimes$ stands for convolution.

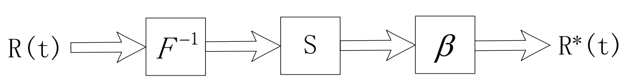

Fig. 2. Regarding the monitoring system as a series of service models.

When the same input data flow corresponds to a variety of processing results, we define the scaling function S as the maximum processing result of the same input flow. The equivalent service curve obtained by this approach is smaller than the real service curve, and the performance boundary of the system obtained in this way can guarantee the real service performance. Furthermore, when the monitoring system has massive simultaneous computation and transmission, any part of the section containing the calculation and the transmission can be equivalent to a computing service node with a transmission service node. If we connect the equivalent service nodes described above in series, according to [26], the equivalent service model can be obtained. Such syncretized equivalent model can be transformed into the traditional transmission service model. The scenario of multi-channel data can be obtained similarly.

In the mode of a sharing network, power system data is required to go through the public network first before going through a private network. We assume that the private network has QoS guarantee for monitoring system. Hence, our focus is put on the analysis of the equivalent transport service model through the public network transmission. Power monitoring data transmission in public networks can be equivalent to competition with the other application data. According to [26], if a service system works on two aggregate flow R_1 and R_2 simultaneously, the overall service curve is $\beta(t)$, and the arrival curves of R_1, R_2 are α_1, α_2, respectively, then, for arbitrary time t the output of R_1 satisfies the following inequality,

$$R_1^*(t) \geq R_1 \otimes (\beta - \alpha_2)^+(t).$$

If $(\beta - \alpha_2)^+$ is a generalized increasing function, then $(\beta - \alpha_2)^+$ is the service curve of R_1. The key to achieve the equivalent transfer service curve is to obtain the arrival curve of the rest of the data in the public network, which is analyzed in the next section.

III. EQUIVALENT TRANSMISSION SERVICE CURVE IN THE SHARED NETWORK

A. Deriving Flow Arrival Curve Based on the Incremental Process Related Functions

According to the properties of fraction, self-similarity and long-range dependence processes, both the fractional dimension describing the fractional characteristics and the Hurst exponent describing the long-range dependence belong to the properties of the process auto-correlation function in a specific range.

Before we provide the models based on the auto-correlation function, a mathematical lemma is presented first.

Lemma I [27] *The variance formula for the sum of arbitrary random variables,*

$$var\left(\sum_{i=1}^{n} X_i\right) = \sum_{i=1}^{n} var(X_i) + 2 \sum_{\substack{i\neq j}}^{i\leq n, j\leq n} cov(X_i, X_j).$$

Based on Lemma I, we denote the network arrival flow as a random stochastic process $A(t)$ which has a finite mean value. We assume that the incremental process is a smooth process, denoted by $F(t)$, and the auto-correlation function is $r(\tau)$. The network flow model is usually assumed to have incremental stability [28]. Otherwise, the network arrival model cannot be defined. Referring to the fractional Gaussian process [29], we define the incremental process similarly,

$$F(t) \triangleq \frac{A(t+\varepsilon) - A(t)}{\varepsilon}.$$

Since the incremental process is stationary, its mean is constant. Thus, we assume $E\big(F(t)\big) = c$. Furthermore, let us define $G(t) \triangleq F(t) - c$.

We assume that the variance of incremental procedure $G(t)$ is σ^2. Since $G(t)$ is a stationary process, σ^2 exists and is constant. Then, we present the main results as follows.

Theorem I: *The variance of arrival flow within any continuous time period* $[0, t]$ *satisfies the following relationship*

$$Var\big(A(t+s)-A(s)\big)=2\sigma^2\int_0^t (t-\tau)r(\tau)d\tau.$$

Proof.

Due to $E\big(G(t)\big)=0$, we have

$$cov(G(t),G(s))=r(t-s).$$

Let us denote $t=n\varepsilon$. Then, we have

$$A(t+s)-A(t+s-\varepsilon)=\varepsilon G[(n-1)\varepsilon+s]+c\varepsilon,$$
$$A(t-\varepsilon+s)-A(t-2\varepsilon+s)=\varepsilon G[(n-2)\varepsilon+s]+c\varepsilon,$$
$$\cdots,$$
$$A(2\varepsilon+\delta+s)-A(\varepsilon+\delta+s)=\varepsilon G(\varepsilon+s)+c\varepsilon,$$
$$A(\varepsilon+s)-A(s)=\varepsilon G(0+s)+c\varepsilon.$$

Let us take the summation of the above n equations, which leads to

$$A(t+s)-A(s)=\textstyle\sum_{i=0}^{n-1}\varepsilon G(i\varepsilon+s)+nc\varepsilon. \quad (1)$$

By Lemma I, the variance of $A(t+s)-A(s)$ can be expressed as

$$\begin{aligned}&Var\big(A(t+s)-A(s)\big)\\&=\varepsilon^2\left[\sum_{i=0}^{n-1}Var(G(i\varepsilon+s)\right.\\&\left.+2\sum_{\substack{i\le n,j\le n\\0<i<j}}cov(G(i\varepsilon+s),G(j\varepsilon+s))\right]\\&=t\sigma^2\varepsilon+2\varepsilon^2\sigma^2\sum_{i=1}^{n-1}(n-i)r(i\varepsilon,\varepsilon).\end{aligned}$$

For $\varepsilon\to 0$, we set $\tau=i\varepsilon$. Then,

$$(n-i)\varepsilon=t-\tau,$$

which leads to $d\tau=\varepsilon$. Then, we have

$$\begin{aligned}Var\big(A(t+s)-A(s)\big)&=\lim_{\varepsilon\to 0}\left[t\sigma^2\varepsilon+2\varepsilon^2\sigma^2\right.\\&\left.\times\sum_{i=1}^{n-1}(n-i)r(i\times\varepsilon,\varepsilon)\right]\\&=\lim_{\varepsilon\to 0}\left[2\varepsilon\sum_{i=1}^{n-1}(n\varepsilon-i\varepsilon)r(i\varepsilon,\varepsilon)\right]=2\sigma^2\int_0^t(t-\tau)r(\tau)d\tau,\end{aligned}$$

which finishes the proof.

□

At initial time 0, we define the input network flow to be 0. Therefore, we can calculate the variance of the flow in any time period according to Theorem I. If the incremental process $G(t)$ satisfies the Gaussian property, then we have one more result as follows.

Theorem II: *Let us define the violation probability*

$$\delta=Pr\big[A(t+s)-A(s)>\hat{A}(t)\big],$$

where $\hat{A}(t)$ *stands for the arrival curve. Given* $k=\sqrt{-2ln\delta}$, *we assume that the incremental process* $G(t)$ *of network flow* $A(t)$ *is a Gaussian stationary process, and the auto-correlation function of* $G(t)$ *is* $r(\tau)$ *with its mean* c *and variance* σ^2, *then the arrival curve* $\hat{A}(t)$ *can be expressed as:*

$$\hat{A}(t)=ct+k\sigma\sqrt{2\int_0^t(t-\tau)r(\tau)d\tau}.$$

Proof.

We recall (1) and define

$$Z(t)\triangleq\sum_{i=0}^{n-1}\varepsilon G(i\varepsilon+s).$$

Then, we have

$$E[Z(t)]=0,$$
$$var[Z(t)]=2\sigma^2\textstyle\int_0^t(t-\tau)r(\tau)d\tau. \quad (2)$$

Due to $nc\varepsilon=ct$, thus,

$$A(t+s)-A(s)=Z(t)+ct.$$

Since $G(t)$ is Gaussian stationary process, a linear combination of stationary Gaussian process remains stationary Gaussian process. $Z(t)$ is a Gaussian process with mean 0 and variance satisfying (2). Then, we have

$$Pr\left\{\frac{Z(t)}{\sqrt{var[Z(t)]}}>k\right\}=\int_k^\infty\frac{1}{\sqrt{2\pi}}e^{-\frac{s^2}{2}}ds.$$

For $k\to\infty$, we have

$$\lim_{k\to\infty}\int_k^\infty\frac{1}{\sqrt{2\pi}}e^{-\frac{s^2}{2}}ds=e^{-\frac{k^2}{2}}.$$

Hence,

$$Pr\left\{\frac{Z(t)}{\sqrt{var[Z(t)]}}>k\right\}=e^{-\frac{k^2}{2}}.$$

Due to $k=\sqrt{-2ln\delta}$, the above equation can be rewritten as

$$Pr\left\{Z(t)>k\sqrt{var[Z(t)]}\right\}=Pr\left\{\frac{Z(t)}{\sqrt{var[Z(t)]}}>k\right\}=\delta.$$

Thus,

$$\begin{aligned}&Pr\left\{A(t+s)-A(s)>k\sqrt{var[Z(t)]}+ct\right\}\\&=Pr\left\{Z(t)+ct>k\sqrt{var[Z(t)]}+ct\right\}\\&=Pr\left\{Z(t)>k\sqrt{var[Z(t)]}\right\}=\delta.\end{aligned}$$

As we are given

$$Pr\big[A(t+s)-A(s)>\hat{A}(t)\big]=\delta,$$

then,

$$\hat{A}(t)=ct+k\sigma\sqrt{2\int_0^t(t-\tau)r(\tau)d\tau}$$

is achieved, which finishes the proof.

□

Based on the idea of solving the above problem in the continuous time case, we present a similar result considered in the discrete time case.

Corollary I: *The overall variance for the discrete time case is expressed by the following equation*

$$var\big(A(n+s)-A(s)\big)=n\sigma^2+2\sigma^2\sum_{i=1}^{n-1}(n-i)r(i).$$

Proof.

If we choose $\varepsilon=1$, then we have the corresponding definition of incremental process in the discrete time case, with

$$G(n)=A(n+1)-A(n).$$

We omit the detailed proof here, which is similar to that of Theorem II.

□

B. Flow Arrival Curve for the case of the Incremental Process being Fractional Gaussian Process

Treating incremental process as fractional Gaussian process is regarded as one of the most commonly used methods to describe the public network flow. Assuming the network flow is fractional Brownian process, in this subsection the flow arrival curve model for the discrete time case is obtained as a theorem. With the auto-correlation function of the discrete time fractional Gaussian process being

$$r(k) = \frac{1}{2}[(k+1)^{2H} + |k-1|^{2H} - 2(k)^{2H}];$$

see, e.g., [30], two technical lemmas are provided before presenting the theorem.

Lemma II *The following equation holds.*

$$\sum_{i=1}^{m}[(i+1)^{2H} + (i-1)^{2H} - 2(i)^{2H}] = (m+1)^{2H} - (m)^{2H} - 1.$$

Proof.

For $m = 1$, we have

$$(2)^{2H} + 0 - 2 = (2)^{2H} - 1 - 1.$$

For $m = r$, we have

$$\sum_{i=1}^{r}[(i+1)^{2H} + (i-1)^{2H} - 2(i)^{2H}] = (r+1)^{2H} - (r)^{2H} - 1.$$

Furthermore, for $m = r+1$, we have

$$\begin{aligned}\sum_{i=1}^{r+1}[(i+1)^{2H} + (i-1)^{2H} - 2(i)^{2H}] &= \sum_{i=1}^{r}[(i+1)^{2H} + (i-1)^{2H} - 2(i)^{2H}] \\ &+ (r+2)^{2H} + (r)^{2H} - 2(r+1)^{2H} \\ &= (r+2)^{2H} - (r+1)^{2H} - 1.\end{aligned}$$

Consequently, we have

$$\sum_{i=1}^{m}[(i+1)^{2H} + (i-1)^{2H} - 2(i)^{2H}] = (m+1)^{2H} - (m)^{2H} - 1,$$

which finishes the proof.

□

Lemma III: *The following equation holds.*

$$\sum_{i=1}^{k}(k+1-i)[(i+1)^{2H} + (i-1)^{2H} - 2(i)^{2H}] = (k+1)^{2H} - (k+1).$$

Proof.

When $k = 1$, the above equation holds obviously. For $k = m$, we have

$$\sum_{i=1}^{m}(m+1-i)[(i+1)^{2H} + (i-1)^{2H} - 2(i)^{2H}] = (m+1)^{2H} - (m+1).$$

For $k = m+1$, we have

$$\begin{aligned}\sum_{i=1}^{m+1}(m+2-i)[(i+1)^{2H} + (i-1)^{2H} - 2(i)^{2H}] &= \sum_{i=1}^{m}(m+1-i)[(i+1)^{2H} + (i-1)^{2H} - 2(i)^{2H}] \\ &+ \sum_{i=1}^{m}[(i+1)^{2H} + (i-1)^{2H} - 2(i)^{2H}] \\ &+ (m+2)^{2H} + (m)^{2H} - 2(m+1)^{2H} = (m+2)^{2H} + (m)^{2H} - (m+1)^{2H} - (m+1) \\ &+ \sum_{i=1}^{m}[(i+1)^{2H} + (i-1)^{2H} - 2(i)^{2H}] \\ &= (m+2)^{2H} + (m)^{2H} - (m+1)^{2H} \\ &- (m+1) + (m+1)^{2H} - (m)^{2H} - 1 \\ &= (m+2)^{2H} - (m+2),\end{aligned}$$

which finishes the proof.

□

Theorem III *The overall flow variance under the discrete time case is expressed by the following equation*

$$var\big(B_H(t+s) - B_H(s)\big) = \sigma^2(n)^{2H}.$$

In addition, the following holds,

$$Pr\{R(t+s) - R(s) > \rho t + k\sigma t^H\} = e^{-\frac{k^2}{2}}. \quad (3)$$

Proof.

According to Lemma II and Lemma III, and assuming that $t = n$, i.e., n times of the unit time, we have

$$\begin{aligned}&Var\big(B_H(t+s) - B_H(s)\big) \\ &= n\sigma^2 + 2\sigma^2 \times \sum_{i=1}^{n-1}(n-i)r(\mathrm{i}) \\ &= n\sigma^2 + \sigma^2 \times \sum_{i=1}^{n-1}(n-i)[(i+1)^{2H} + (i-1)^{2H} - 2(i)^{2H}] \\ &= n\sigma^2 + \sigma^2 \times [(n)^{2H} - n] \\ &= \sigma^2(n)^{2H}.\end{aligned}$$

According to Theorem II, we have equation (3), which finishes the proof.

□

C. Flow Arrival Curve for the case of the Incremental Process being Generalized Cauchy Process

In Section III-A, we have discussed the flow arrival curve when the incremental process is fractional Gaussian, but there is a linear relationship between fractional dimension and Hurst exponent of fractional Gaussian [31,32], namely,

$$D + H = 2.$$

This implies that fractional Gaussian process is related with the local properties (fractional) and global characteristics (length correlation), while an intuitive consideration of this relevance will cause defects. The generalized Cauchy process is defined as follows.

Definition I *If a random process is stationary Gaussian process, and its auto-correlation function satisfies the following equation:*

$$r(\tau) = E\big(X(t+\tau)X(t)\big) = (1+|\tau|^{\alpha})^{-\frac{\beta}{\alpha}},$$

in which $0 < \alpha \leq 2$, $\beta > 0$, *such stationary Gaussian process is known as the Generalized Cauchy process.*

It can be shown that

$$r(\tau) = 1 - \frac{\beta}{\alpha}|\tau|^{\alpha},$$

for $|\tau| \to 0$. Thus,

$$r(0) - r(\tau) \sim \frac{\beta}{\alpha}|\tau|^{\alpha},$$

for $|\tau| \to 0$. Meanwhile,

$$r(\tau) = |\tau|^{-\beta},$$

for $|\tau| \to \infty$. Hence,

$$D = 2 - \frac{\alpha}{2},$$
$$H = 1 - \frac{\beta}{2}.$$

Thereby, in the generalized Cauchy process, D is no longer related with H. However, the auto-correlation function itself is not sufficient to describe the network flow. In fact, the fractional dimension is a criterion for the local self-similarity; see, e.g., [33]. Based on local self-similarity, the network flow arrival model via fractional dimension and long correlation coefficient is proposed in [34]. Two parameters a and r are introduced in their model, and it is notable that these two parameters are not measurable at present. Hence, these models are unable to be used in practical applications. According to Theorem II, we present the network flow arrival model under the generalized Cauchy hypothesis:

$$\hat{A}(t) = ct + k\sigma\sqrt{2\int_0^t (t-\tau)(1+|\tau|^{\alpha})^{-\frac{\beta}{\alpha}}d\tau},$$

with

$$Pr\big[A(t+s) - A(s) > \hat{A}(t)\big] = \delta$$

and

$$k = \sqrt{-2ln\delta}$$

being satisfied. One cannot obtain the analytical result with the term

$$\sqrt{2\int_0^{\mathrm{t}} (\mathrm{t}-\tau)(1+|\tau|^{\alpha})^{-\frac{\beta}{\alpha}}\mathrm{d}\tau},$$

whereas an approximation solution can be achieved by numerical algorithm. When modelling the network flow practically, since the arrival flow is defined as the discrete process in the minimum time scale, we are able to obtain the flow arrival curve by Corollary I.

IV. Experimental Results

A. Experimental Design

In this section, we verify the arrival flow curves obtained in Section III-C according to the actual network flow dataset, and we compare the experimental results with the arrival flow curves obtained in Section III-B. In order to highlight the importance of this experiment, we adopt the network flow dataset in [35], which is widely used in related research filed. The experimental steps are as follows:

Step 1. Given time scale (millimeter, second, etc.), we record the size of the network data in per unit time (s). The data in a unit time can be regarded as the incremental process of the flow process at the current scale.

Step 2. Focusing on the incremental process obtained by the first step, we calculate the fractional dimension D, Hurst exponent H, mean ρ and standard deviation σ, respectively. Among a variety of approaches, we adopt the variable diagram method [36] to calculate the fractional dimension and the detrended fluctuation analysis (DFA) method [37] to calculate the Hurst exponent. In this section, the *RandomFields* software package of the R language is used, which can also be used to obtain the fractional dimension and Hurst exponent. Next, we obtain the fractional Gaussian process and the generalized Cauchy process as,

$$r_{fgn}(k) = \frac{1}{2}[(k+1)^{2H} + |k-1|^{2H} - 2(k)^{2H}],$$

and

$$r_{GC}(\tau) = E\big(X(t+\tau)X(t)\big) = (1+|\tau|^{\alpha})^{-\frac{\beta}{\alpha}},$$

respectively.

Step 3. According to the auto-correlation functions of the fractional Gaussian process and the generalized Cauchy process, the variance of the network flow for a given time t is as follows:

$$var_{fgn}\big(A(t)\big) = \sigma^2(n)^{2H},$$

and

$$var_{GC}\big(A(t)\big) = n\sigma^2 + 2\sigma^2 \times \sum_{i=1}^{n-1}(n-i)(1+n^{\alpha})^{-\frac{\beta}{\alpha}}.$$

Step 4. We treat the given time period t as the unit time, count the network data, find its variance, and compare it with the variance obtained in Step 3.

Step 5. Given the error δ, the flow arrival curves for the case of fractional Gaussian process and generalized Cauchy process in a given time t are as follows:

$$A_{fgn}(t) = \rho n + k\sigma n^{H},$$

and

$$A_{GC}(t) = \rho n + k\sqrt{n\sigma^2 + 2\sigma^2 \times \sum_{i=1}^{n-1}(n-i)(1+n^{\alpha})^{-\frac{\beta}{\alpha}}}.$$

Step 6. We calculate the size of the maximum network data for the real data and that of the maximum data package under the given error during time interval t.The size of the maximum data package under the given error refers to the maximum value excluding θ numbers, where θ satisfies

$$\theta = \delta \times \omega.$$

Here, ω stands for the total data sample. We compare the computation results with the ones obtained in the Step 5.

Step 7. When t increases subject to multiple of s, we repeat Step 4, Step 5, Step 6. Then, we draw the curves for both actual and theoretical data.

B. Experimental Results

We set the time unit as 0.001 second. With data in [35], we obtain the statistics of the fractional dimension D, Hurst exponent H, mean ρ and standard deviation σ. The results are as follows:

$$\rho = 138.1854,$$
$$\sigma = 1.1575 \times 10^5,$$
$$H = 0.8573,$$
$$D = 1.9205.$$

According to the experimental procedures in Section IV-A, diagrams with the curves of the incremental process are plotted, in which Curve 1 refers to the calculation results obtained under the assumption of fractional Gaussian process, Curve 2 refers to the calculation results obtained under the assumption of generalized Cauchy process, and Curve 3 refers to the statistical results from real data.

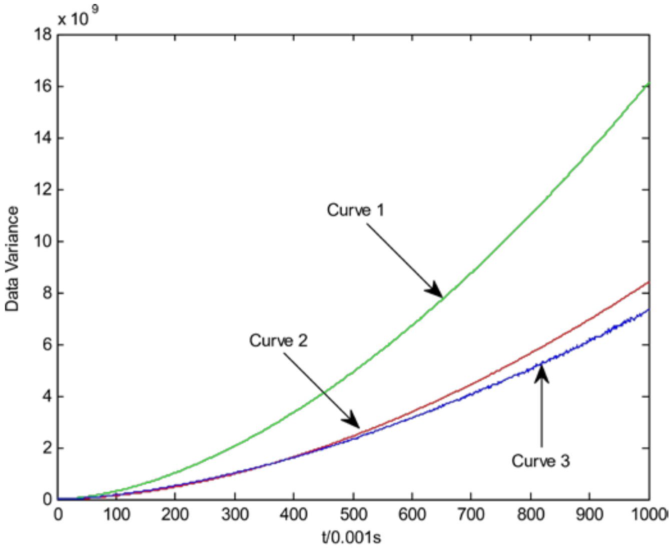

Fig. 3. Data variance (unit time 0.001s).

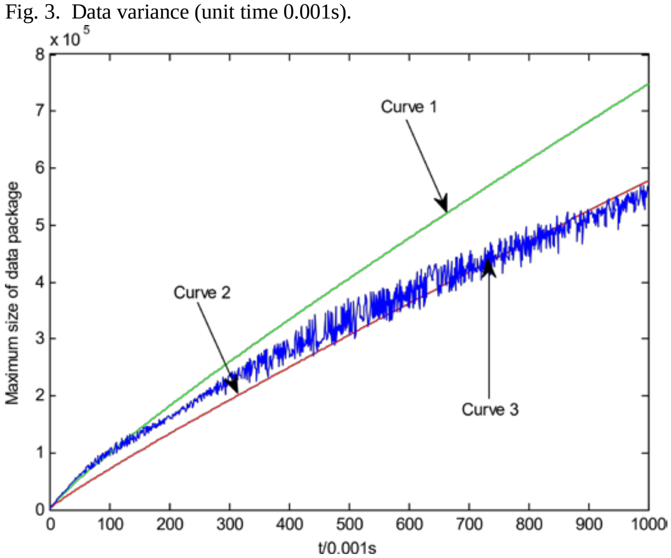

Fig. 4. Maximum data package (unit time 0.001s).

Fig. 3 shows the comparison of the variance between two different models and that of the actual data when the experimental time is multiples of 0.001s. In Fig. 4, the experimental time is still multiples of 0.001s and we set δ to be 0.0001, focusing on the maximum data package. Fig. 5 focuses on the size of the maximum data package under a given error δ.

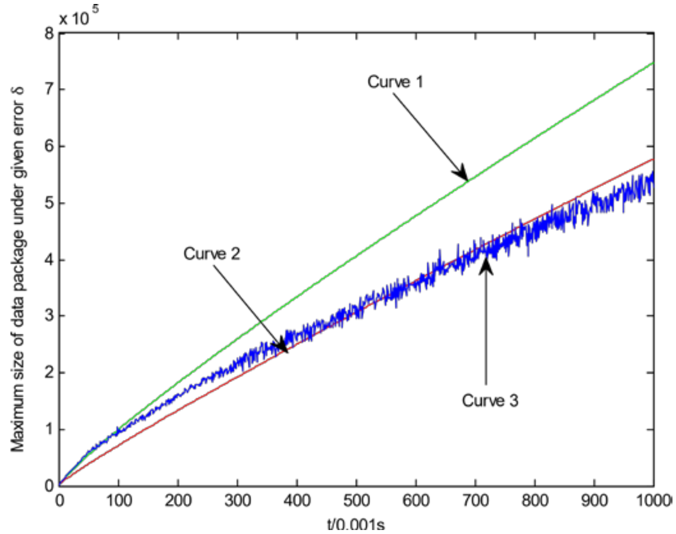

Fig. 5. The size of the maximum data package under a given error δ (unit time 0.001s).

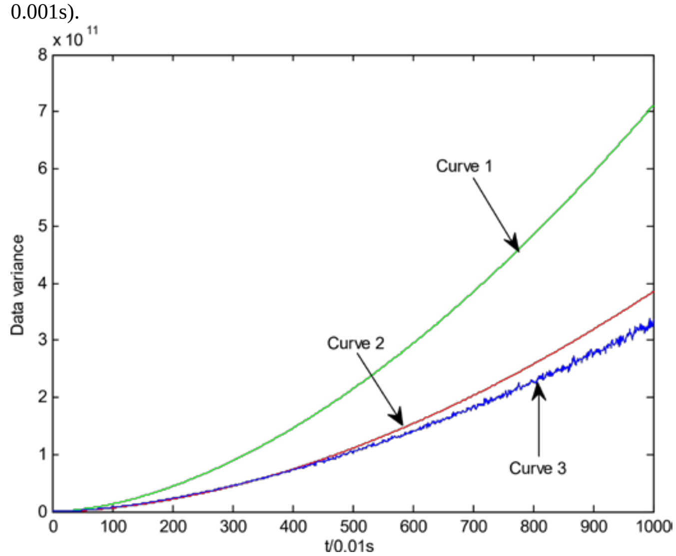

Fig. 6. Data variance (unit time 0.01s).

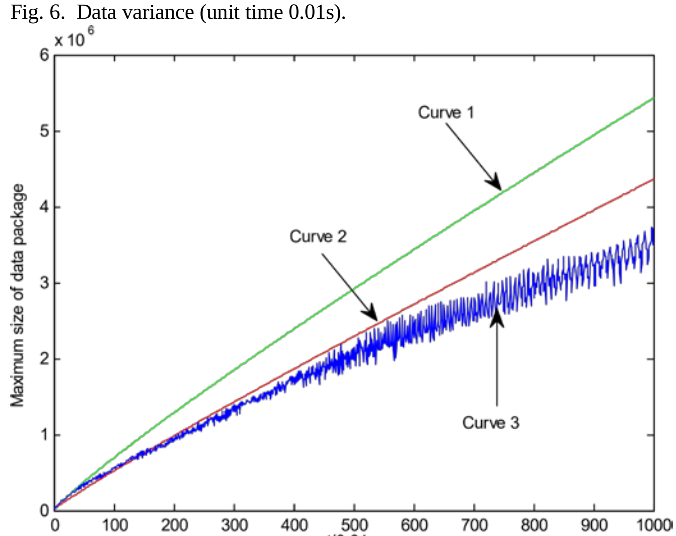

Fig. 7. The size of maximum data package (unit time 0.01s).

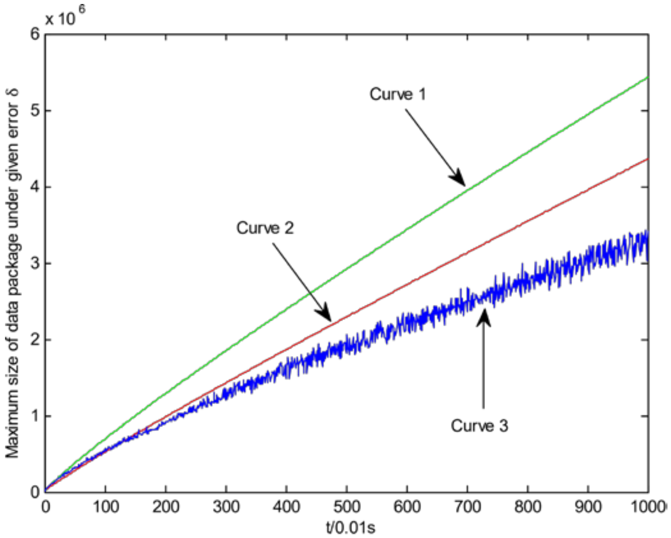

Fig. 8. The size of the maximum data package under a given error δ (unit time 0.01s)

Fig. 6, Fig. 7 and Fig. 8 are plotted for the case of the unit time being 0.01s. From Fig. 3 to Fig. 8, it is obvious that the flow arrival curves obtained by the methodology proposed in this paper is feasible, and the calculation results obtained under generalized Cauchy process is closer to the statistical results from real data than that of fractional Gaussian process. In summary, the assumption of generalized Cauchy process appears to be more appropriate.

V. SUMMARY

In this paper, we study the equivalent model of monitoring system service performance. Attentions are drawn to the calculation problem of the equivalent transmission services curve under the shared network model. We propose the method of obtaining the node flow arrival curves based on the auto-correlation functions. Several lemmas, theorems and corollaries are obtained from the mathematical perspective. Base on the real data, different models are verified via simulations. The results indicate that generalized Cauchy process performs better than fractional Gaussian process for the considered problem. It is highlighted that we are focusing on developing theoretical results, and the analysis methods introduced in this paper can be applied under most scenarios of monitoring systems in smart grid.

In recent years, the concept of energy Internet has been popular and is regarded as the 2.0 version of smart grid. In the field of energy Internet, energy and information are fused, and a bottom-up energy management principle is expected to be achieved by making full use of the information exchange of the entire system [38]. The latency problem of communication systems considered in this paper also exists in the energy Internet system. In addition, the studied performance modelling for data monitoring services can be further extended to energy transmission systems in energy Internet scenarios. For our future research, research on modelling and control for communication systems in energy Internet scenarios will be investigated.

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Junwei Cao (SM'05) received his Ph.D. in computer science from the University of Warwick, Coventry, UK, in 2001. He received his bachelor and master degrees in control theories and engineering in 1998 and 1996, respectively, both from Tsinghua University, Beijing, China. He is currently Professor and Vice Dean of Research Institute of Information Technology, Tsinghua University, Beijing, China. He is also Director of Open Platform and Technology Division, Tsinghua National Laboratory for Information Science and Technology.

Prior to joining Tsinghua University in 2006, he was a Research Scientist at MIT LIGO Laboratory and NEC Laboratories Europe for about 5 years. He has published over 200 papers and cited by international scholars for over 18,000 times. He has authored or edited 8 books. His research is focused on distributed computing technologies and energy/power applications.

Prof. Cao is a Senior Member of the IEEE Computer Society and a Member of the ACM and CCF.

Haochen Hua (M'16) was born in Jiangsu, P. R. China in 1988. He received the B.Sc. degree in mathematics with finance in 2011, and the Ph.D. degree in mathematical sciences in 2016, both from the University of Liverpool, Liverpool, UK.

He is currently an assistant professor in the Research Institute of Information Technology, Tsinghua University, Beijing, P. R. China. His current research interests include optimal and robust control theory and its applications in power systems, smart grids, and the energy internet.