Short-Term Energy Cache Regulation for Energy Router: A Robust H-infinity Approach

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***Abstract*—With the development and evolution of power system, the penetration of renewable energy increases continuously. The energy Internet is proposed as the future of conventional power systems, and it ensures the utilization efficiency of renewable energy sources and promises more reliable and intelligent energy services. Being able to transfer and cache energy-information flows among microgirds, energy routers play the key role in the construction of future energy Internet. It is important to ensure the stability of the core energy transfer functionalities of energy routers. In this paper, the short-term energy cache regulation for energy routers is investigated. A robust control method against the potential uncertainties in energy transmission processes is proposed. The energy cache regulation efficacy of the proposed control method is evaluated with numerical simulation. The results demonstrate the feasibility and effectiveness of the proposed method.**

***Index Terms*—robust control, energy router, energy internet**

I. Introduction

With the growing concerns on climate change and environment contamination, the development of renewable energy and relevant industries have received extensive attentions. However, most of these renewable energy sources, such as solar power and wind power, are highly relevant to environment conditions, i.e., solar irradiation, precipitation, wind speed, etc. Affected by the uncertainties in their deployment environment, these renewable energy sources are of the stochastic, intermittent and uncertain natures, which make it difficult to integrate them in the conventional power system directly.

For better integration and utilization of renewable energy resources, the concept of microgrid was firstly proposed by the consortium for electric reliability technology solutions (CERTS) in 2002 [1], [2]. A microgrid is a small power system consisting of renewable energy sources, generators, energy storage and relevant loads [1]. The smart meters and intelligent control system deployed in a microgrid enable more precise and sensible energy management [2], which helps to improve the utilization efficiency of the integrated renewable energy generators. The microgrid is proved to be a successful structure for the renewable energy integration in the conventional power utility [3], and microgrid related researches are carried out extensively [4]–[6].

While the penetration of renewable energy increase, it becomes difficult to keep the balance between power service qualities and energy efficiency within a microgrid. The smart grid is proposed as a solution to tackle these difficulties [7], [8]. With various sensors deployed, the smart grid solution allows power-sharing in large area as well as the coordination of multiple microgrids, and it is able to send notifications to customers via the Internet, which engages users in the loop of energy management. Further, in future energy Internet, multiple relevant sectors, such as heating, fuel supply and transportation systems, are included in the scope of the energy Internet, and more features of the Internet are introduced [8]–[10]. With advances in communication and sensing technologies, comprehensive scheduling and management of multiple systems in energy Internet becomes possible, which allows more intelligent and reliable energy services in energy Internet.

The energy Internet could be built up in a bottom-up manner [8]. In this sense, distributed local power grids, i.e., mostly microgrids, are interconnected via energy routers, and the energy-information flow are transferred, cached and traded through the energy routers network [8]. The distributed and flexible characteristics of energy Internet greatly improve the resilience and service quality of the whole system, and energy routers play a key role in the realization of these core features [11].

Plenty of the research outputs related to energy Internet are focus on the power management issue in microgrids. In [4], [6], [12], [13], the frequency regulation issues of microgrids are investigated. Similarly, the voltage stabilization approach for islanded microgrid is introduced in [14]. For the better utilization of renweable energy, the coordinated operation for multiple microgrid systems are studied in [5], [15]. In [16], [17], stochastic processes are used to model the uncertainties in the power output of renewable energy sources such as photovoltaic panels and wind turbines. The power estimation technologies for power consumption are introduced in [17], [18], as well. Based on the advanced modeling methods mentioned above, in [19], deep reinforcement learning approach are applied to achieve the power management in a regional energy Internet system.

There are also some literature relevant to energy routers in the energy Internet, see, e.g., [13], [20], [21]. The functionality of energy routers in microgirds are taken into consideration in [13], [20], such that the energy management principle of energy Internet could be realized. On the other hand, the energy routing strategies in energy Internet are developed in [21], which provides solutions for the scheduling and management of energy router networks.

In this paper, we focus on the short-term energy transfer and cache management issue of energy routers. Based on the control theory, a kind of robust H_∞ control method is proposed for the short-term energy cache regulation for energy routers.

The rest of this paper is organized as follows. Firstly, in Section II, the short-term model for energy cache in energy routers is introduced. Then the energy cache regulation task is formulated as a robust H_∞ control problem in Section III, and the solution for the control problem is provided in Section IV. The effectiveness and feasibility of the proposed control method are evaluated in Section V. Finally, Section VI summarizes this paper.

II. System Modeling

Energy routers are important infrastructures in the energy Internet. At the microgrid level, energy routers provide versatile accesses for the renewable energy sources and various electrical equipment, such that the renewable energy resources including photovoltaic panels and wind turbines can be utilized more efficiently. At the same time, the energy router interconnects the microgrids in a regional energy Internet, which enables the energy trading/sharing in a large area.

In this paper, short term energy cache management issue related to the power routing functionality of energy routers is investigated. The concept and features of energy router is inspired by routers in Internet. An energy router is able to transfer the energy-information flows among microgrids as well as buffering the flows with low priority. The buffered energy flows would be cached in the energy storage devices integrated in energy routers. With limited energy caching capacity, the amount of power flow buffered should be maintained in proper range. Especially, for the short term management of energy routers, the size of energy cache is expected to be regulated in the preset working point, such that the energy transmission network built upon energy routers could be stable in daily operations.

Here, taking the assumption that the transmission protocols deployed in energy Internet are similar as the transmission control protocol (TCP) in Internet, based on the fluid model for TCP network flows [22], [23], the energy routing in an energy router can be modeled as follows.

$$\dot{W}(t) = \frac{1}{R(t)} - \frac{W(t)W(t-R(t))}{2R(t)}p(t-R(t)), \tag{1}$$

$$\dot{q}(t) = \frac{W(t)N(t)}{R(t)} - C(t), \tag{2}$$

where $W(t)$ is the window size for energy transmission; $q(t)$ is the instantaneous amount of energy cache in the energy router; $R(t)$ is the round trip time of the power and information exchange for active energy links; $C(t)$ is the energy transmission capability of the considered energy router; $N(t)$ is the number of active energy-information routing links; $0 \le p(t) \le 1$ is the dropping probability for the energy transmission requests.

Based on the energy routing model in (1) and (2), by adjusting the dropping probability $p(t)$, the size of energy cache in an energy router could be regulated accordingly. The dropping probability $p(t)$ in (1) is used to selectively reject the transmission requests received by the energy router. If a transmission request is rejected, the corresponding energy flow would not be transferred to the considered energy router. In this sense, when the size of energy cache get close to the capacity of the energy router, a high dropping probability would help to restrict the increase of energy cache, and vice versa.

In this paper, we focus on the short term energy cache regulation of the energy router. Supposing that the energy router is working at the set point (W_0, q_0, p_0), by applying linearization techniques [22], [23], the nonlinear model in (1) and (2) can be approximated by (3) and (4) in short periods.

$$\begin{aligned}\Delta\dot{W}(t) = & -\frac{2N_0}{R_0^2C_0}(\Delta W(t)+\Delta W(t-R_0)) - \frac{1}{R_0^2C_0}(\Delta q(t) \\ & -\Delta q(t-R_0)) - \frac{R_0C_0^2}{2N_0^2}\Delta q(t-R_0),\end{aligned} \tag{3}$$

$$\dot{q}(t) = \frac{N_0}{R_0}\Delta W(t) - \frac{1}{R_0}\Delta q(t), \tag{4}$$

where $\Delta W = W(t) - W_0$, $\Delta q = q(t) - q_0$, $\Delta p(t) = p(t) - p_0$. The term R_0, N_0 and C_0 refers to the round trip time, number of activated energy links and energy transmission capability at time $t = 0$, respectively.

Let us denote $X(t) = [\Delta W(t), \Delta q(t)]'$, $u(t) = \Delta p(t)$, the linearized model in (3) and (4) could be rewritten in (5)

$$\begin{cases}\dot{X}(t) = AX(t) + A_dX(t-R_0) + Bu(t), \\ Z(t) = CX(t).\end{cases} \tag{5}$$

In the real world applications, the energy transmission capability, round trip time and number of active links of the energy router are varying continuously. To take these complex factors into consideration, parameter uncertainties, disturbance inputs and time-varying delay are introduced in (5). The improved model is given in (6).

$$\begin{cases}\dot{X}(t) = (A+\Delta A)X(t) + (A_d+\Delta A_d)X(t-d) \\ \qquad +B_1\omega(t) + (B_2+\Delta B_2)u(t), \\ Z(t) = CX(t).\end{cases} \tag{6}$$

where $Z(t)$ is an observation for the system state $X(t)$; $\omega(t)$ is the disturbance input; time-varying delay d satisfies $R_0 \le d \le R_0 + h$. Here, h refers to the delay jitter, which would be affected by the real-time communication quality changes. The system parameters in (6) are shown as follows.

$$A = \begin{bmatrix} -\frac{N_0}{R_0^2C_0} & -\frac{1}{R_0^2C_0} \\ \frac{N_0}{R_0} & -\frac{1}{R_0}\end{bmatrix}, A_d = \begin{bmatrix} -\frac{N_0}{R_0^2C_0} & -\frac{1}{R_0^2C_0} \\ 0 & 0\end{bmatrix},$$

$$B_2 = \begin{bmatrix} -\frac{R_0C_0^2}{2N_0^2} \\ 0\end{bmatrix}, C = \begin{bmatrix} 0 & 1\end{bmatrix}.$$

The corresponding parameter uncertainty matrices ΔA, ΔB_2 and ΔA_d are modeled with

$$[\Delta A \quad \Delta B_2 \quad \Delta A_d] = E\Sigma(t)[F_1 \quad F_2 \quad F_d],$$

where $\Sigma(t)'\Sigma(t) \leq 1$.

III. Problem Formulation

With the system model (6) in Section II, the energy cache management problem for energy routers are formulated as follows.

As stated previously, in this paper, we focus on the short term energy cache regulation task of the investigated energy router. One of the most important concerns is the stability of the energy transfer system. More specifically, the stability of the size of energy cache. Once the energy cache deviates its reference value in an energy router, the operation of other connected routers would be potentially affected. Local malfunctions triggered by the instability of individual energy routers may lead to the failure of the whole energy Internet. In this sense, it is quite important to deploy appropriate control schemes for energy routers.

In this paper, the L_2 norm based H_∞ performance defined in (7) is used to measure the stability of energy cache size in an energy router.

$$J = \|Z(t)\|_2 - \gamma^2\|\omega(t)\|_2 \leq 0, \tag{7}$$

where γ is a preset disturbance attenuation factor; $Z(t)$ and $\omega(t)$ are observed energy cache size and system disturbance in (6), respectively; $\|\cdot\|_2$ is the L_2 norm. For a given function $f(t)$, its L_2 norm is calculated as follows.

$$\|f(t)\|_2 = \int_0^\infty f(t)'f(t)\mathrm{d}t. \tag{8}$$

The value of γ in (7) reflects the strictness of the H_∞ performance, and it should be set accordingly based on the practical situations. Generally, an H_∞ performance with smaller γ would be more difficult to be satisfied.

The robust H_∞ control problem for the energy routers is then formulated as follows.

$$J \leq 0$$
$$\begin{cases} \dot{X}(t) = (A + \Delta A)X(t) + (A_d + \Delta A_d)X(t-d) \\ \qquad +B_1\omega(t) + (B_2 + \Delta B_2)u(t), \\ Z(t) = CX(t). \end{cases} \tag{9}$$

IV. Robust Control Approach

In this section, two theorems are introduced to obtain desired controller for the robust H_∞ control problem (9).

Firstly, Theorem 1 provides a sufficient condition for the asymptotic stability of system (6).

Theorem 1 ([22]): Given a feedback controller $u(t) = KX(t)$, supposing that the disturbance input $\omega(t) = 0$, if there exist symmetric semidefinite matrices P, Q, M, Z, matrix N and positive scalar $\varepsilon > 0$, such that the matrix inequality (10) holds, where $*$ refers to transpose of the corresponding element in matrix (10), $\bar{d} = R_0 + h$,

$$\Omega = P(A + B_2K) + (A + B_2K)' + dM + N + N',$$

then the system is asymptotically stabilized with controller $u(t) = KX(t)$.

Theorem 1 shows that the system (6) could be stabilized with specific controllers without external disturbances. But the controller cannot be obtained directly from (10). Based on Theorem 1, Theorem 2 indicates the sufficient condition for controllers that ensure the robust stability of the considered energy router system. The sufficient condition is formulated as a linear matrix inequality, which can be solved without essential difficulty.

Theorem 2 ([22]): With the disturbance input $\omega(t)$ and attenuation factor γ, if there exist symmetric semi-definite matrices X, Q, M, Z, matrices Y, N and positive scalar $\varepsilon > 0$, such that the linear matrix inequality (11) holds, where $*$ refers to transpose of the corresponding element in matrix (11), $\bar{d} = R_0 + h$,

$$\Xi = AX + B_2Y + XA' + Y'B_2' + N + N' + Q,$$

then the system is asymptotically stabilized with controller $u(t) = KX(t)$ and the H_∞ performance (7) is satisfied. Here, $K = YX^{-1}$.

With the two theorems introduced above, the solution for the robust H_∞ control problem (9) is obtained.

V. Simulation

To evaluate the effectiveness of the proposed robust H_∞ control approach, numerical simulation results are provided in this section.

In the simulations, the disturbance input $\omega(t)$ for system (6) is approximated with Ornstein-Uhlenbeck process [17] defined in (12).

$$\mathrm{d}\omega(t) = -\theta(\omega(t) - \mu)\mathrm{d}t + \sigma\mathrm{d}w(t), \tag{12}$$

where θ, μ and σ are parameters characterizing the disturbance input $\omega(t)$; $w(t)$ is a standard scalar Wiener process. The time period of the simulation is set to be $t \in [0, 3s]$, and the parameters used in simulation is provided in Table I.

TABLE I
Parameters for Simulation

Parameter	Value	Parameter	Value
N_0	10	C_0	1500
R_0	$0.23(s)$	h	$0.12(s)$
γ	1.3	ε	1.9
θ	3.8	μ	0.5
σ	1.0	$\bar{d}$	$3.5(s)$

During simulation, coefficient for disturbance input $\omega(t)$ and parameter uncertainties matrices in (6) are given as follows.

$$B_1 = \begin{bmatrix} 10 \\ 5 \end{bmatrix}, E = \begin{bmatrix} 1.0 & 0.3 \\ -0.2 & 1.0 \end{bmatrix},$$

$$\begin{bmatrix} \Omega & -N+PA_d & (A+B_2K)' & PE & (F_1+F_2K)' \\ * & -Q & A_d' & 0 & F_d' \\ * & * & -\bar{d}^{-1}Z & E & 0 \\ * & * & * & -\varepsilon I & 0 \\ * & * & * & * & -\varepsilon^{-1}I \end{bmatrix} \leq 0 \tag{10}$$

$$\begin{bmatrix} \Xi & A_dX-N & B_1 & XA'+Y'B_2' & M & XC' & E & XF_1'+Y'F_2' \\ * & -Q & 0 & XA_d' & 0 & 0 & 0 & XF_d' \\ * & * & -\gamma^2 & B_1' & 0 & 0 & 0 & 0 \\ * & * & * & -\bar{d}^{-1}Z & 0 & 0 & E & 0 \\ * & * & * & * & -\bar{d}^{-1}M & 0 & 0 & 0 \\ * & * & * & * & * & -I & 0 & 0 \\ * & * & * & * & * & * & -\varepsilon I & 0 \\ * & * & * & * & * & * & * & -\varepsilon^{-1}I \end{bmatrix} \leq 0 \tag{11}$$

$$F_1 = \begin{bmatrix} 0.185 & 0.185 \\ 0.317 & 0.317 \end{bmatrix}, F_2 = \begin{bmatrix} 0.264 \\ 0.053 \end{bmatrix},$$

$$F_d = \begin{bmatrix} 0.264 & 0.264 \\ 0.053 & 0.053 \end{bmatrix}.$$

Based on the system parameters provided above, the linear matrix inequality in (11) is solved with cvxpy [24], [25], which is a Python convex optimal toolbox. The obtained feedback gain $K = [0.5423 \quad 0.1272]$.

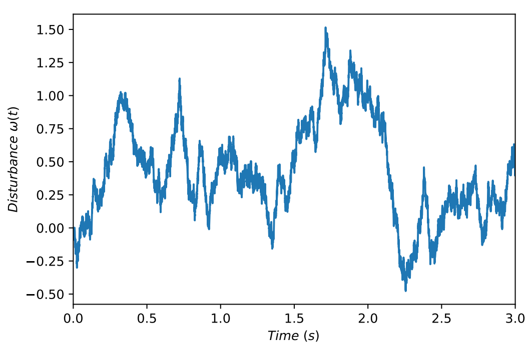

Fig. 1. Disturbance input $\omega(t)$.

The disturbance input $\omega(t)$, which is generated from (12), during simulation is shown in Fig. 1. The time delay for the considered energy router is plotted in Fig. 2. It can be found that the generated time-varying delay is within the upper and lower bounds, i.e., $R_0 + h$ and R_0.

The deviations of the parameter uncertainties, i.e., $\Sigma(t)$, are illustrated in Fig. 3

With the disturbance input in Fig. 1, the delay of energy-information flow of energy router in Fig. 2 and the parameter uncertainty deviations in Fig. 3, the controller $u(t) = KX(t)$ is applied to the energy router system (6).

The main simulation results are shown in Fig. 4, Fig. 5 and Fig. 6, respectively. The deviations of dropping probability for transmission requests are shown in Fig. 4. It is easy to figure

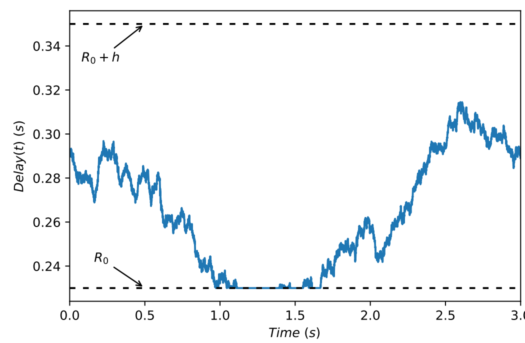

Fig. 2. Time delay in the considered energy router.

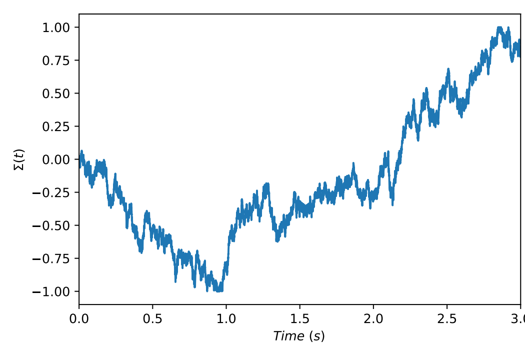

Fig. 3. Deviations of parameter uncertainties.

out that the deviations of dropping probability are kept within a small range. Thus, the overall performance of the energy flow transmission via the considered energy router will not be influenced significantly by the energy cache regulation.

The deviations of W(t), i.e., energy transmission window of the investigated energy router, are shown in Fig. 5. The curve of $\Delta W(t)$ under the obtained robust H_∞ controller is illustrated with solid line. For comparison, trajectory of $\Delta W(t)$ without energy cache regulation is plotted with dashed line.

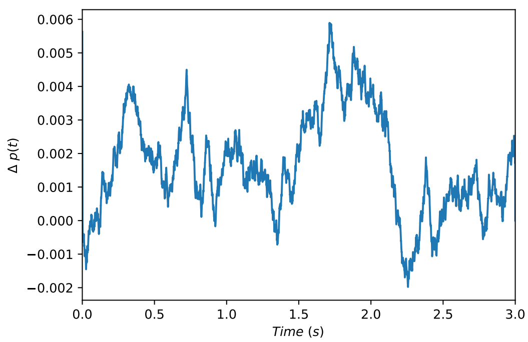

Fig. 4. Dropping probability deviations $\Delta p(t)$.

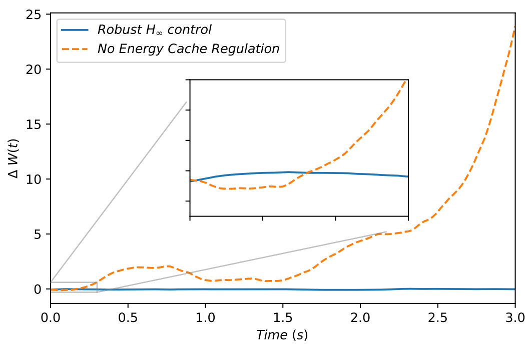

Fig. 5. Fluctuations of energy flow transmission window.

Similarly, curves of energy cache fluctuations under proposed controller and that without energy cache regulation are plotted in Fig. 6, with solid and dashed lines, respectively. It is obvious that with the proposed robust H_∞ controller, the magnitude of transmission window and the energy cache size are perfectly maintained around the set point. When there is no energy cache regulation performed, the working point of the energy router would deviate the set point within 0.3s, which would lead to the failure of the linearized system (6) and incur potential damages to the energy routers and energy transmission network.

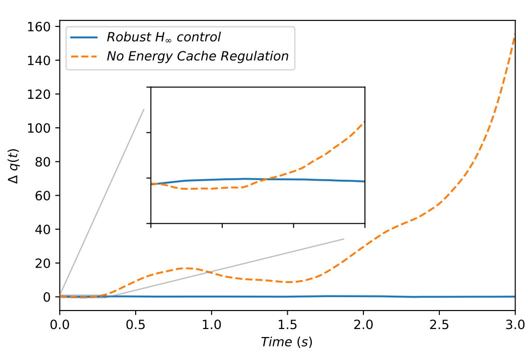

Fig. 6. Changes in energy cache size $\Delta q(t)$.

Based on the simulation result above, the effectiveness and feasibility of the proposed control approach are evaluated. The simulation results demonstrate the importance of energy cache regulation for energy routers.

VI. Conclusion

In this paper, the energy cache regulation problem for energy routers in the energy Internet is investigated. With the linearized energy transmission model, a kind of robust H_∞ controller for short-term energy cache management is proposed. The efficacy and feasibility of the proposed control method are successfully evaluated with numerical simulations.

Since the long-term dynamics of the energy flows in the energy Internet and the influences from microgrids are not fully considered, Our future research would focus on the further integration of energy routers in different energy Internet scenarios.

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References

[1] R. Lasseter, A. Akhil, C. Marnay, J. Stephens, J. Dagle, R. Guttromson, A. Meliopoulous, R. Yinger, and J. Eto, "The CERTS microgrid concept," *White paper for Transmission Reliability Program, Office of Power Technologies, US Department of Energy*, vol. 2, no. 3, p. 30, 2002.

[2] R. H. Lasseter and P. Piagi, "Microgrid: A conceptual solution," in *Proc. IEEE Power Electronics Specialists Conference*, vol. 6. Citeseer, 2004, pp. 4285–4291.

[3] A. Q. Huang, M. L. Crow, G. T. Heydt, J. P. Zheng, and S. J. Dale, "The future renewable electric energy delivery and management (FREEDM) system: the energy internet," *P. IEEE*, vol. 99, no. 1, pp. 133–148, 2011.

[4] H. Hua, Y. Qin, and J. Cao, "A class of optimal and robust controller design for islanded microgrid," in *Proc. 2017 IEEE 7th International Conference on Power and Energy Systems (ICPES)*. IEEE, 2017, pp. 111–116.

[5] X. Fang, Q. Yang, J. Wang, and W. Yan, "Coordinated dispatch in multiple cooperative autonomous islanded microgrids," *Appl. Energ.*, vol. 162, pp. 40–48, Jan. 2016.

[6] H. Bevrani, M. R. Feizi, and S. Ataee, "Robust frequency control in an islanded microgrid: H_∞ and μ-synthesis approaches," *IEEE Trans. Smart Grid*, vol. 7, no. 2, pp. 706–717, Mar. 2016.

[7] J. W. Eising, T. V. Onna, and F. Alkemade, "Towards smart grids: Identifying the risks that arise from the integration of energy and transport supply chains," *Appl. Energ.*, vol. 123, pp. 448–455, Jun. 2014.

[8] J. Cao and M. Yang, "Energy internet–towards smart grid 2.0," in *Proc. 2013 Fourth International Conference on Networking and Distributed Computing*. IEEE, 2013, pp. 105–110.

[9] N. Bui, A. P. Castellani, P. Casari, and M. Zorzi, "The internet of energy: A web-enabled smart grid system," *IEEE Network*, vol. 26, no. 4, pp. 39–45, 2012.

[10] J. Cao, H. Hua, and G. Ren, "Energy use and the Internet," in *The SAGE Encyclopedia of the Internet*, 2018, pp. 343–350.

[11] Y. Xu, J. Zhang, W. Wang, A. Juneja, and S. Bhattacharya, "Energy router: Architectures and functionalities toward energy internet," in *Proc. 2011 IEEE International Conference on Smart Grid Communications (SmartGridComm)*. IEEE, 2011, pp. 31–36.

[12] Y. Han, P. M. Young, A. Jain, and D. Zimmerle, "Robust control for microgrid frequency deviation reduction with attached storage system," *IEEE Trans. Smart Grid*, vol. 6, no. 2, pp. 557–565, Mar. 2015.

[13] H. Hua, C. Hao, Y. Qin, and J. Cao, "A class of control strategies for energy Internet considering system robustness and operation cost optimization," *Energies*, vol. 11, no. 6, p. 1593, 2018.

[14] L. Sedghi and A. Fakharian, “Robust voltage regulation in islanded microgrids: A LMI based mixed H_2/H_∞ control approach,” in *Proc. 24th Mediterranean Conf. on Control and Automation*. Athens, Greece: IEEE, Jun. 2016, pp. 431–436.

[15] H. Hua, Y. Qin, and J. Cao, “Coordinated frequency control for multiple microgrids in energy Internet: A stochastic H_∞ approach,” in *Proc. 2018 IEEE Innovative Smart Grid Technologies-Asia (ISGT Asia)*. IEEE, 2018, pp. 810–815.

[16] M. Miozzo, D. Zordan, P. Dini, and M. Rossi, “SolarStat: Modeling photovoltaic sources through stochastic Markov processes,” in *Proc. IEEE Int. Energy Conf.* Dubrovnik, Croatia: IEEE, May 2014, pp. 688–695.

[17] H. Hua, Y. Qin, C. Hao, and J. Cao, “Stochastic optimal control for energy Internet: A bottom-up energy management approach,” *IEEE Trans. Ind. Informat.*, vol. 15, no. 3, pp. 1788–1797, Mar. 2018.

[18] J. L. Mathieu, S. Koch, and D. S. Callaway, “State estimation and control of electric loads to manage real-time energy imbalance,” *IEEE Trans. Power Syst.*, vol. 28, no. 1, pp. 430–440, Feb. 2013.

[19] H. Hua, Y. Qin, C. Hao, and J. Cao, “Optimal energy management strategies for energy Internet via deep reinforcement learning approach,” *Appl. Energ.*, vol. 239, pp. 598–609, Apr. 2019.

[20] H. Hua, Y. Qin, J. Geng, C. Hao, and J. Cao, “Robust mixed H_2/H_∞ controller design for energy routers in energy Internet,” *Energies*, vol. 12, no. 3, p. 340, 2019.

[21] R. Wang, J. Wu, Z. Qian, Z. Lin, and X. He, “A graph theory based energy routing algorithm in energy local area network,” *IEEE Trans. Ind. Informat.*, vol. 13, no. 6, pp. 3275–3285, Dec. 2017.

[22] P. Sun, *Robust H_∞ control theory and applications*, 1st ed. Tsinghua University Press, 2012.

[23] B. Marami and M. Haeri, “Implementation of MPC as an AQM controller,” *Comput. Commun.*, vol. 33, no. 2, pp. 227–239, 2010.

[24] S. Diamond and S. Boyd, “CVXPY: A Python-embedded modeling language for convex optimization,” *J. Mach. Learn. Res.*, vol. 17, no. 83, pp. 1–5, 2016.

[25] A. Agrawal, R. Verschueren, S. Diamond, and S. Boyd, “A rewriting system for convex optimization problems,” *J. Control Decis.*, vol. 5, no. 1, pp. 42–60, 2018.